

Flow Patterns, Log Mean Temperature Difference, Approach Temperatures, and Temperature Crosses in Sensible Heating and Cooling Applications

There are many equations that are used in the rating and design of heat exchangers; some are empirical and some are based on principles of physics and chemistry. Two common equations for heat exchangers in single phase steady state flow are;

[1] $Q = \dot{m} c_p (T_i - T_o)$, where;

Q represents heat load in BTU/hr
 $\dot{m}$ represents mass flow rate in lbs./hr
 c_p represents Specific Heat in BTU/lb-°F (constant - specific to the fluid)
 T_i represents fluid inlet temperature in °F
 T_o represents fluid outlet temperature in °F

goto; <http://www.deltathx.com/uploadsDocs/HT201.pdf>, for "quick approximations" of this equation with common fluids

[2] $Q = U A [LMTD]$, where;

Q represents heat load in BTU/hr
 U represents the overall heat transfer coefficient in lbs./hr-ft²-°F
 A represents the heat transfer surface area in ft²
[LMTD] represents the Log Mean Temperature Difference in °F

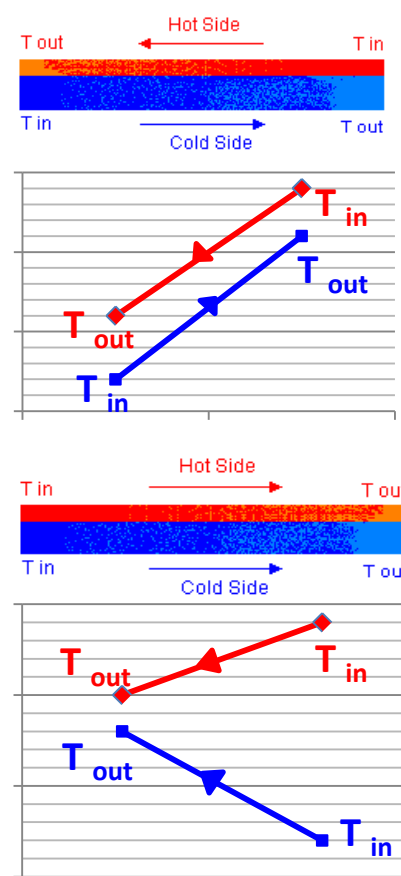
These equations interact in a related way that can help us assess the process and perhaps improve or optimize the heat transfer system. Equation [1] relates to the physical change in the fluid as a result of the heat transfer system. Equation [2] is system related and takes into consideration specifics of the system such as flows and heat transfer surface area.

Counter Flow, Parallel Flow and Cross Flow

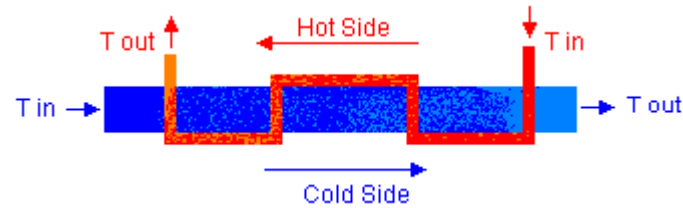
The flow pattern of the two fluids in a heat exchanger can vary depending on the heat exchanger design and type. Typically the process conditions, available space and economics will define the type and configuration of the heat exchanger.

The most effective configuration from a heat transfer standpoint is a **counter flow** design (also called counter current). In a counter flow arrangement the hot fluid and cold fluid move in opposite directions, such that , at one terminal point; the hot side inlet temperature (hottest) is in contact with the cold side outlet temperature (hottest) and at the other terminal point; the cold side inlet temperature (coldest) is in contact with the hot side outlet temperature (coldest). Counter flow designs are most valuable when there is a temperature cross. A temperature cross occurs when the desired outlet temperature of one fluid is between the inlet and outlet temperatures of the other fluid. A true counter flow design is difficult to achieve or impractical for many common heat exchanger applications, however close to true counter flow can be obtained using plate heat exchangers, double pipe exchangers and spiral heat exchangers.

The reverse configuration to counter flow would be **parallel flow**, where the hot fluid and cold fluid move in the same direction, such that, at one terminal point; the hot side inlet temperature (hottest) is in contact with the cold side inlet temperature (coldest), the two fluid progress through the heat exchanger and at the other terminal point; the cold side outlet temperature (hottest) is in contact with the hot side outlet temperature (coldest). A temperature cross cannot be achieved with a parallel flow exchanger. Parallel flow configuration is frequently used in applications where it is desired that the two fluids have close to the same exiting temperatures.



Most shell and tube heat exchangers and air-cooled heat exchangers are of **cross flow** design because it can sometimes be impractical to use a counter flow design due to high flow volumes or space restrictions. The flow patterns two fluids in a cross flow heat exchanger are at right angles to each other. In most applications, at least one fluid makes multiple passes so there is a combination of cross flow and counter flow occurring. With the thermal advantage gained from a multi-pass configuration a temperature cross can be produced in cross flow heat exchanger but not to the same degree as in a true counter flow configuration.

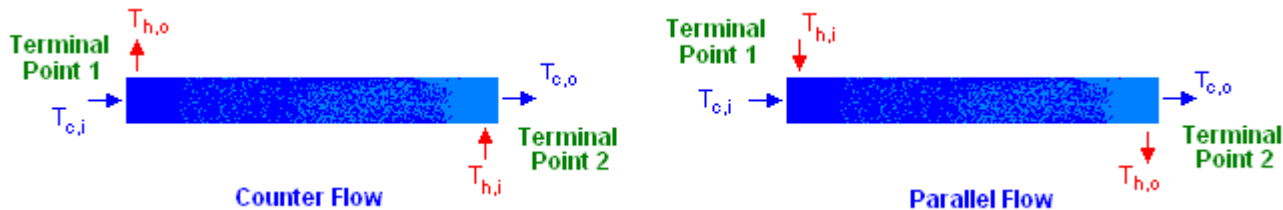


Log Mean Temperature Difference

The log mean temperature difference (LMTD) is a logarithmic average of the temperature difference between the hot and cold streams at each end of the exchanger. The larger the LMTD, the more heat is transferred. To understand the relevance of the concept of the LMTD, consider that hot oil can be cooled much more effectively with cold water than with warm water. The LMTD is calculated by the following equation;

$$[LMTD] = \frac{\Delta T_1 - \Delta T_2}{\ln[\Delta T_1/\Delta T_2]}, \text{ where } \Delta T_1 \text{ and } \Delta T_2 \text{ are the temperature differences at the two terminal points.}$$

The terminal points are locations at the heat exchanger where the two fluids either enter or leave the heat exchanger. It is important to distinguish between terminal points and entrances and exits because depending on whether the heat exchanger configuration is counter flow, cross flow or parallel flow, the entrances and exits on one side may be reversed.



So for a counter flow application; $\Delta T_1 = T_{h,o} - T_{c,i}$ and $\Delta T_2 = T_{h,i} - T_{c,o}$

And for a parallel flow application; $\Delta T_1 = T_{h,i} - T_{c,i}$ and $\Delta T_2 = T_{h,o} - T_{c,o}$

To demonstrate the advantage of counter flow over parallel flow, consider a heat exchanger designed to cool 30 usgpm of water from 150°F to 98°F using 35 usgpm of 50/50 water/glycol at 40°F. If a true counter flow heat exchanger is used, the LMTD would be; (using equation [2] the outlet temperature of the glycol is determined to be 92.7°F)

$$LMTD = [\Delta T_1 - \Delta T_2] / \ln[\Delta T_1/\Delta T_2] = [(98 - 40) - (150 - 92.7)] / \ln[(98 - 40) / (150 - 92.7)] = 57.7^\circ\text{F}$$

If a true parallel flow heat exchanger is used, the LMTD would be;

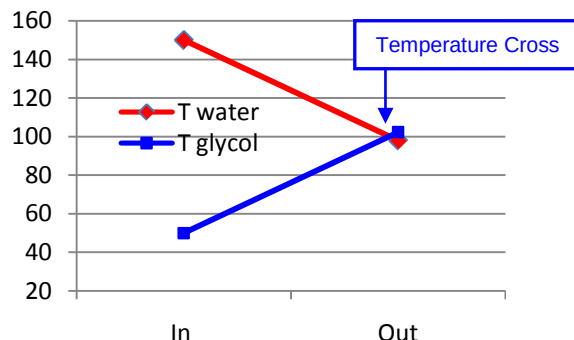
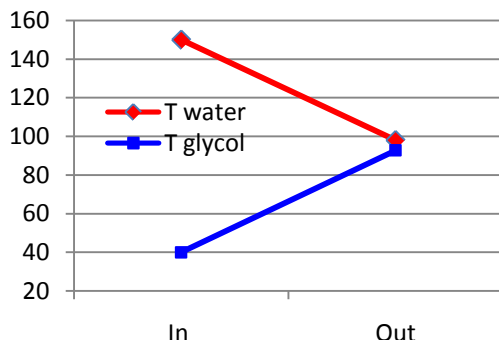
$$LMTD = [\Delta T_1 - \Delta T_2] / \ln[\Delta T_1/\Delta T_2] = [(150 - 40) - (98 - 92.7)] / \ln[(150 - 40) / (98 - 92.7)] = 34.5^\circ\text{F}$$

Applying Equation [2], the difference in the LMTD would need to be made up in overall heat transfer coefficient (which is difficult to do) or by increasing the surface area of the heat exchanger.

The LMTD is the variable in equation [2], which factors the effect of the temperature profiles of the two fluids involved in the heat exchanger. To demonstrate how the [approach temperatures](#) affect the LMTD, consider the last example, except the 50/50 water/glycol inlet temperature is 50°F. In a true counter flow configuration, the LMTD would be; (using equation [2] the outlet temperature of the glycol is determined to be 102.3°F)

$$LMTD = [\Delta T_1 - \Delta T_2] / \ln[\Delta T_1/\Delta T_2] = [(98 - 50) - (150 - 102.3)] / \ln[(98 - 50) / (150 - 102.3)] = 47.9^\circ\text{F}$$

The difference in LMTD between a coolant temperature of 40°F and 50°F is 20% which for practical purposes, would need to be made up by increasing the overall heat transfer co-efficient or by increasing the surface area of the heat exchanger.



Corrected Mean temperature Difference

As discussed above, it is difficult to achieve true counter flow in many heat exchanger applications, therefore, a correction factor (F) is often applied to the LMTD so it may be used in equation [2]. A corrected LMTD is often referred to on a specification sheet as the CMTD. The correction factor is determined by such factors as flow pattern, terminal temperatures and heat release curve. For a non-counter flow heat exchanger the LMTD shown on the specification sheet is usually the Corrected LMTD.

For example, consider a shell and tube heat exchanger designed to cool 30 usgpm of water using 35 usgpm of 50/50 water/glycol at 40°F. Let us first consider that the water is to be cooled from 150°F to 98°F.

30 usgpm of water $\approx$ 14,855 lbs/hr

35 usgpm of 50/50 water/glycol $\approx$ 18,662 lbs/hr

C_p water = 0.999 BTU/lb-°F

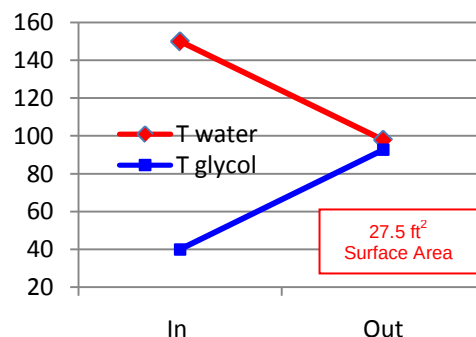
C_p 50/50 water/glycol = 0.785 BTU/lb-°F

Using equation [1] for the water side,

$Q = 14,855 \times 0.999 \times (150 - 98) = 771,688$ BTU/Hr.

There must be a heat balance, so on the glycol side;

$Q = 771,688 = 18,662 \times 0.785 \times (T_{out} - 40)$, so $T_{out} = 92.7^\circ\text{F}$.



The temperature profile of the process is shown in the graph on the right. Note that the glycol outlet temperature is less than the water outlet temperature. The temperature rise (ΔT) of the water glycol is 52.7°F and the ΔT on the water side is 52°F

We calculate the CMTD = 48.7°F and through a complex calculation related to the specific heat exchanger design and fluid properties we calculate $U = 576.34$ lbs./hr-ft²-°F. Now applying equation [2], we can calculate the heat transfer surface area required for the heat exchanger;

$Q = U A [\text{LMTD}]$, or $A = Q / \{U \times [\text{LMTD}]\} = 771,688 / \{576.34 \times 48.7\} = 27.5$ ft² of surface area is required

In most cases, surface area is directly related to the heat exchanger cost. To demonstrate the impact of approach temperatures on the surface area, consider the above application except the inlet glycol temperature is now 35°F. At a constant flow, the ΔT of the water glycol is still 52.7°F therefore the glycol outlet temperature is now 87.7°F.

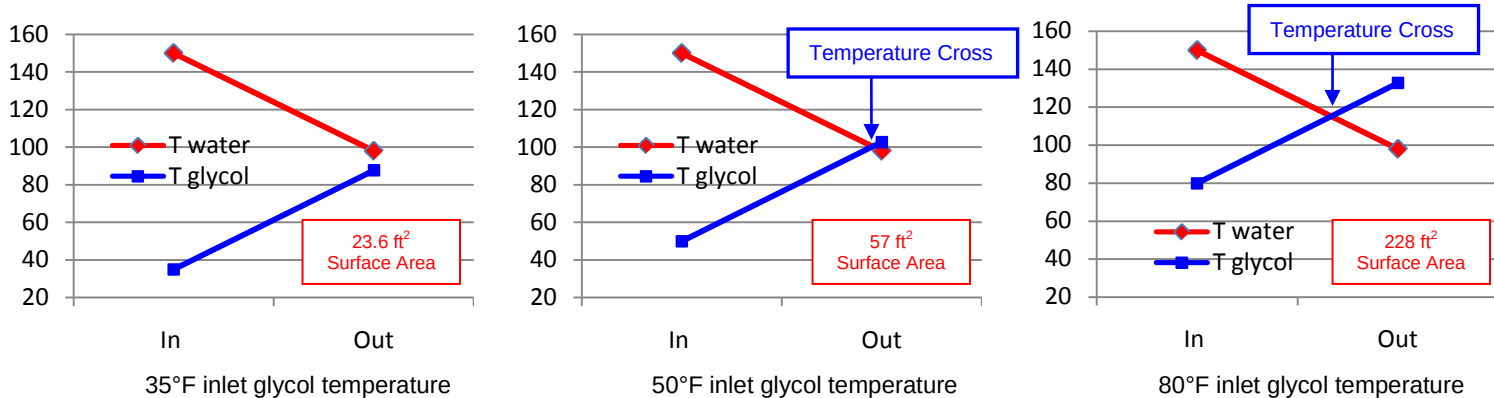
Under the 35°F glycol inlet temperature, we calculate the CMTD = 54.5°F and $U = 601.29$ lbs./hr-ft²-°F.

$A = Q / \{U \times [\text{LMTD}]\} = 771,688 / \{601.29 \times 54.5\} = 23.6$ ft² of surface area is required (14% less surface area)

Now consider an inlet glycol temperature of 50°F. At a constant flow, the ΔT of the water glycol is still 52.7°F, therefore the glycol outlet temperature is now 102.7°F.

Under the 50°F glycol inlet temperature, we calculate the CMTD = 36.2°F and $U = 374.25 \text{ lbs./hr-ft}^2\text{-}^\circ\text{F}$.

$A = Q / \{U * [LMTD]\} = 771,688 / \{374.25 * 36.2\} = 57 \text{ ft}^2$ of surface area is required (107% more surface area)



In this application, the highest inlet glycol temperature to achieve a practical cross flow design is 80°F, which results in a 132.7°F glycol outlet temperature, 18.4°F CMTD and $U = 183.9 \text{ lbs./hr-ft}^2\text{-}^\circ\text{F}$. The surface area required for this design is 228 ft² (almost 10 times the required surface if the inlet glycol temperature were 35°F).

Approach Temperatures

The above example demonstrates the impact that the LMTD has on the size (and therefore cost) of a cross flow, sensible cooling application. To get a broad concept of the viability of a particular sensible cooling application, the LMTD does not necessarily need to be calculated, just the approach temperatures considered. The approach temperatures are the difference between the Outlet Temperature of one stream and the Inlet Temperature of the other stream. Although each application will have two approach temperatures, typically it is obvious which one is important from a design standpoint. For most heat exchanger applications three of the four terminal temperatures are fixed, so the fourth temperature will be determined by equation [1]. From a practical design standpoint, it might be important to consider the approach temperature that involves the stream that can be controlled; for example, if the flow of coolant can be increased to allow a lower outlet temperature, or if the inlet temperature of the cooling stream can be reduced – these will both increase the approach temperatures resulting in less surface area in the heat exchanger.

In a sensible cooling or heating application, obviously the outlet temperature of the cold stream cannot be higher than the inlet temperature of the hot stream nor can the outlet temperature of the hot stream be lower than the inlet temperature of the cold stream. (Equation [1] can be helpful in determining if the application is possible)

Temperature Cross

We have seen from some of the above examples that a temperature cross can be achieved in a cross flow or counter flow design and we have also seen that a temperature cross does have an effect on the efficiency of heat transfer. In many applications a temperature cross cannot be avoided and the alternatives are to use multiple exchangers or to use exchangers that are as close to true counter flow as possible (such as; spiral, [plate heat exchangers](#), [double pipe](#), or [hairpin exchangers](#)).